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THE VECTOR AND AXIAL-VECTOR DOMINANCE OF WEAK INTERACTIONS

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The problem of the vector and axial-vector dominance of weak interactions in the framework of the quark model of superconductivity type is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Векторная и аксиально-векторная доминантность слабых взаимодействий

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Обсуждается проблема векторной и аксиально-векторной доминантности слабых взаимодействий в рамках кварковой модели сверхпроводящего типа.

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The question how to include the weak interactions into the quark model of superconductivity type (QMST) /1/ has been discussed in papers /2,3/ Here we would like to give the final variant of such an inclusion. It will be slightly different from the results, obtained in that part of /3/, where the interaction of axial-vector mesons with W-boson was described. If one uses the approximative mass formula $m^2_{a_1} = m^2_\rho + 6m^2_u$ which takes place in our model, the results /3/ will coincide with those, obtained in this article. However, for the concrete physical calculations it is better to use the last variant, as a more exact one.

After these general remarks let us overcome to the derivation of formulae, describing the vector and axial-vector dominance in QMST. At the beginning we write the Lagrangian, describing in the framework of our model the interaction of mesons with quarks

$$L_{st} = \bar{q}[\, i\hat{\partial} - M + g(\bar{\sigma} + i\gamma^5\bar{\phi}) + \frac{g_\rho}{2}(\hat{\bar{V}} + \gamma^5\hat{\bar{A}})\,]\, q -$$

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$$-\frac{\sigma_a^2+\phi_a^2}{2G_1}+\frac{V_a^2+A_a^2}{2G_2}, \qquad (1)$$

where $\bar{q}=(\bar{u},\bar{d},\bar{s})$ are the quark fields with three colours; $i\hat{\partial}=i\gamma^{\mu}\partial_{\mu}$; $M=\mathrm{diag}(m_u, m_d, m_s)$ is the constituent quark mass matrix; σ_a, ϕ_a, V_a^{μ} and A_a^{μ} are the scalar, pseudoscalar, vector and axial-vector mesons, respectively; $\bar{a}\equiv a_a\lambda^a$ $(0<a<8)$, λ^a are the Gell-Mann matrices; $g_\rho=\sqrt{6}g$ is the constant of decay $\rho\to 2\pi$, $(g_\rho^2/4\pi\approx 3)$. The constant G_1 and G_2 can be determined from the mass spectrum of pseudoscalar and vector mesons: $G_1=4.9\ (\mathrm{GeV})^{-2}$ and $G_2=16\ (\mathrm{GeV})^{-2}$ /1/.

From (1) one can obtain the following (necessary for the further investigations) part of "strong" Lagrangian:

$$L'_{st}=i\sqrt{2}g\bar{q}\gamma^5\pi^+\tau_+q+\frac{g_\rho}{\sqrt{2}}\bar{q}(\hat{\rho}^++\gamma^5\hat{a}_1^+)\tau_+q+g_\rho F_\pi Z a^+_{1\mu}\partial_\mu\pi^-+\mathrm{h.c.}-$$

$$-\frac{1}{2}(\rho^+_{\mu\nu}\rho^-_{\mu\nu}+a^+_{1\mu\nu}a^-_{1\mu\nu})+m_\rho^2\rho^+_\mu\rho^-_\mu+m^2_{a_1}a^+_{1\mu}a^-_{1\mu}, \qquad (2)$$

where $F_\pi=93$ MeV is the pion decay constant. The factor Z can be expressed in terms of physical quantities as follows

$$Z^{-1}=(1-\frac{6m_u^2}{m^2_{a_1}})=\frac{1}{2}[1+\sqrt{1-(\frac{2g_\rho F_\pi}{m^2_{a_1}})^2}\,], \qquad (3)$$

where m_{a_1} is the mass of the a_1-meson. The mass of u-quark is given by the formula

$$m_u^2=\frac{m^2_{a_1}}{12}[1-\sqrt{1-(\frac{2g_\rho F_\pi}{m^2_{a_1}})^2}\,] \qquad (4)$$

and for $m_{a_1}=1200$ MeV we have $m_u\approx 300$ MeV.

Further let us investigate the interaction of the electroweak gauge boson W with quarks and leptons. We write the "weak" Lagrangian in the next form:

$$L_w=-\frac{\kappa}{2\sqrt{2}}\{\bar{q}\hat{W}^{(+)}(1-\gamma^5)\tau_+q+\ell^{\mu(-)}W_\mu^{(+)}+\mathrm{h.c.}\}-$$

$$-\frac{1}{2} W^{(-)\mu\nu} W^{(+)}_{\mu\nu} + M^2_w W^{(-)\mu} W^{(+)}_{\mu} , \tag{5}$$

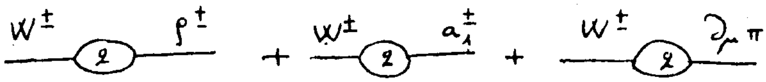
where

$$\ell^{\mu(-)} = \bar{\nu}_e \gamma^\mu (1-\gamma^5) e + \bar{\nu}_\mu \gamma^\mu (1-\gamma^5) \mu + \bar{\nu}_\tau \gamma^\mu (1-\gamma^5) \tau$$

is the lepton current, W_μ is the vector boson with the mass M_w, $W^{(-)\mu\nu} W^{(+)}_{\mu\nu}$ is the kinetic form of W-boson and κ is a weak constant.

The Lagrangians (2) and (5) lead to an appearance of the diagrams depicted in the Figure, corresponding to the nondiagonal terms in

$$\Delta L = -\frac{\kappa}{2} F_\pi Z W^\mu \partial_\mu \pi^- + \frac{\kappa}{4g_\rho} [W^{\mu\nu +} (\rho^-_{\mu\nu} + a^-_{\mu\nu}) - 12 m^2_u W^{\mu +} a_{1\mu}] + \text{h. c.} \tag{6}$$



Figure

At the evaluation of the divergent diagrams in the Figure, we used the relation /1/

$$g = (4I)^{-1/2}, \quad I = -i \frac{3}{(2\pi)^4} \int d^4 q \frac{\theta(\Lambda^2 - q^2)}{(m^2 - q^2)^2} , \tag{7}$$

where at the regularization of divergent integral, playing an important role at the description of quark loops, we used the cut-off parameter Λ. This parameter bounds the region, where the spontaneous breaking of chiral symmetry takes place /1/.

We perform the diagonalization of kinetic terms by taking the new variables:

$$\rho^\pm_\mu = \rho'^\pm_\mu + \frac{\kappa}{2g_\rho} W^\pm_\mu ; \qquad a^\pm_{1\mu} = a'^\pm_{1\mu} + \frac{\kappa}{2g_\rho} W^\pm_\mu . \tag{8}$$

This procedure eliminates the vertices from the Lagrangians L_w and ΔL, describing the interaction of the W-boson with quark fields, as well as the nondiagonal terms $W^{\mu\pm} \partial_\mu \pi^\pm$. The nondiagonal terms with derivations are replaced by the similar terms without derivations and the total Lagrangian will have the form

$$L = L_{st} - \frac{1}{2} W^{(-)\mu\nu} W^{(+)}_{\mu\nu} + M^2_w W^{(-)\mu} W^{(+)}_{\mu} +$$

$$+ \frac{\kappa}{2 g_\rho} [m^2_\rho W^+_\mu \rho^-_\mu + \frac{m^2_{a_1}}{Z} W^+_\mu a^-_{1\mu} + \text{h.c.}] - \frac{\kappa}{2\sqrt{2}} [\ell^{\mu(-)} W^+_\mu + \text{h.c.}] . \tag{9}$$

In the limit of large masses M_W the Lagrangian L can be rewritten in the expression

$$L_w = \frac{G_w \cos\theta_c}{g_\rho} \ell^{\mu(-)} (m^2_\rho \rho^+_\mu + \frac{m^2_{a_1}}{Z} a^+_{1\mu}) + \text{h.c.} \tag{10}$$

Here θ_c is the Cabbibo angle. The vector and axial-vector dominance of weak interactions in QMST has been firstly discussed in papers /2, 3/. In /3/ the factor $m^2_\rho (\rho^+_\mu + a^+_{1\mu})$ has been obtained instead of that one in (9), i.e. the Lagrangian in /3/ has the form

$$L_w = \frac{G_w \cos\theta_c \, m^2_\rho}{g_\rho} \ell^{\mu(-)} (\rho^+_\mu + a^+_{1\mu}) + \text{h.c.} \tag{10a}$$

One can rewrite the Lagrangian (10) into the form (10a) by using the mass formula $m^2_a = m^2_\rho + 6 m^2_u$ and (8), defined in our model

$$m^2_\rho \rho^+_\mu + (1 - \frac{6m^2_u}{m^2_{a_1}}) \, m^2_a \, a^+_{1\mu} = m^2_\rho (\rho^+_\mu + a^+_{1\mu}) .$$

However, because the model formula $m^2_{a_1} = m^2_\rho + 6m^2_u$ leads to the low value of the mass of the a_1-meson ($m_{a_1} \approx 1100$ MeV) it is better to use the Lagrangian of the form given in (10). We verify it by considering, e.g., the decay $\tau \to \nu a_1$. From (10) we get the decay width

$$\Gamma_{\tau\to\nu a_1} = \frac{m_\tau}{8\pi} [G \cos\theta_c \frac{m_{a_1} m_\tau}{Z g_\rho}]^2 (1 - \frac{m^2_{a_1}}{m^2_\tau}) (1 + \frac{2m^2_\rho}{m^2_\tau}) =$$

$$= \begin{cases} 3 \times 10^{-10}\ \text{MeV} \quad (m_{a_1} = 1265\ \text{MeV})\ /4/ \\ 2.7 \times 10^{-10}\ \text{MeV} \quad (m_{a_1} = 1200\ \text{MeV})\ /5/ \end{cases} \tag{11}$$

$$\Gamma^{exp}_{\tau \to \nu (a_1 \to 3\pi)} = (2.85 \pm 0.3 \pm 0.34) \times 10^{-10}\ \text{MeV}\ /4/ .$$

By taking the Lagrangian in the form (10a) we obtain

$$\Gamma_{\tau \to \nu a_1} = \begin{cases} 0.81 \times 10^{-10}\ \text{MeV} \quad (m_{a_1} = 1265\ \text{MeV}) \\ 1.08 \times 10^{-10}\ \text{MeV} \quad (m_{a_1} = 1200\ \text{MeV}) . \end{cases} \tag{12}$$

We write also the theoretical result for the decay $\tau \to \nu \rho$:

$$\Gamma_{\tau \to \nu \rho} = \frac{m_\tau}{8\pi} \left[G \cos\theta_c \frac{m_\rho m_\tau}{g_\rho} \right]^2 \left(1 - \frac{m_\rho^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right) = 4.2 \times 10^{-10}\ \text{MeV}, \tag{13}$$

$$\Gamma^{exp}_{\tau \to \nu \rho} = (4.35 \pm 0.4 \pm 0.53) \times 10^{-10}\ \text{MeV}\ /4/ .$$

There the agreement with the experiment is good. The axial-vector dominance allows to simplify the calculations in any other processes, e.g., the decays $\pi \to e\nu\gamma$ /6/, $\pi \to \nu e (\ell^+ \ell^-)$ /7/, $K_{\ell 4}$ /8/ , etc.

In conclusion let us discuss how to describe the fundamental decay of pion $\pi^- \to e\bar{\nu}(\mu\bar{\nu})$ by using the Lagrangian (10). The corresponding part of (10) has the form

$$\Delta L_w = G_w \cos\theta_w \frac{m_{a_1}^2}{g Z} a^+_{1\mu} \bar{\nu}\gamma^\mu (1 - \gamma^5) e . \tag{10b}$$

To be able to overcome to the physical fields $a'_{1\mu}$, it is necessary to remove the nondiagonal terms in the strong Lagrangian (2). For this purpose it is enough to perform the following transformation of the fields $a_{1\mu}$

$$a^+_{1\mu} = a'^+_{1\mu} - \frac{g_\rho F_\pi Z}{m_{a_1}^2} \partial_\mu \pi^+ , \tag{14}$$

Then one can easily obtain, using (10b) and (14), the Lagrangian describing the decay $\pi^- \to e\bar{\nu}$

$$L_{\pi^- \to e\bar{\nu}} = -G_w \cos\theta_c \left[\frac{m^2_{a_1}}{g_\rho Z} \frac{g_\rho F_\pi Z}{m^2_{a_1}} = F_\pi \right] \partial_\mu \pi^+ \bar{\nu} \gamma^\mu (1 - \gamma^5)\, e =$$

$$= -G_w \cos\theta_c F_\pi \partial_\mu \pi^+ \bar{\nu} \gamma^\mu (1 - \gamma^5)\, e \,. \qquad (15)$$

We have demonstrated that the axial vector dominance plays an important role at the calculation of weak processes.

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